Mathematics grade 8

Polygons

Sum of interior angles of a polygon

Triangle (3-sided Polygon)

In previous grades, you learnt that the sum of interior angles of a triangle is 180^0.
Consider Figure 1.

$$a + b + c = 180^0.$$

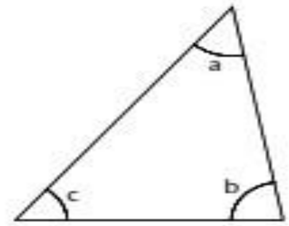
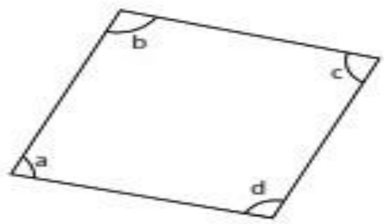


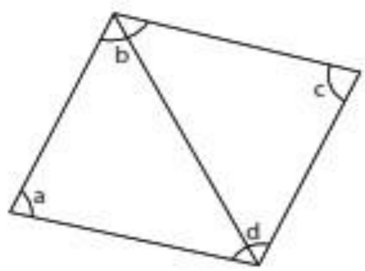
Figure 1

Quadrilateral (4-sided polygon)

Consider the quadrilateral shown.



This quadrilateral can be divided into two triangles as shown below.



Since the sum of interior angles of a triangle is 180^0 and the quadrilateral has 2 triangles, the sum of all the interior angles of the quadrilateral is

$$a + b + c + d = 2 \times \text{sum of interior angles of triangle}$$

$$a + b + c + d = 2 \times 180^0$$

$$a + b + c + d = 360^0$$

Hence, the **sum** of the **interior angles** of a **quadrilateral** is **360^0**.

Example 1

Find the sum of the interior angles of a polygon with 15 sides.

Solution

Number of sides (n) = 15.

Number of triangles = 15 − 2 = 13

Sum of interior angles $= (n - 2) \times 180^0$

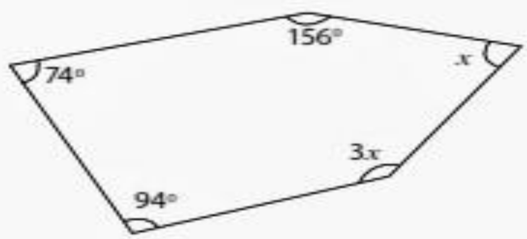
$= (15 - 2) \times 180^0$

$= 13 \times 180^0$

$= 2\,340^0$

Example 2

Find the value of x in the diagram.



Solution

$n = 5$

Number of triangles = 5 − 2 = 3

Sum of interior angles $= (5 - 2) \times 180^0$

$= 3 \times 180^0$

$= 540^0$

Thus,

$$x + 3x + 94^0 + 74^0 + 156^0 = 540^0$$

$$4x + 324^0 = 540^0$$

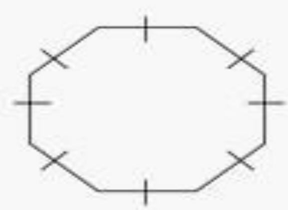
$$4x = 540^0 - 324^0$$

$$4x = 216^0$$

$$x = \frac{216^0}{4} = 54^0$$

Example 3

Find the size of one interior angle of a **regular** octagon.



Solution

Number of sides in an octagon (n) = 8.

Sum of interior angles of an octagon $= (n - 2) \times 180^0$

$= (8 - 2) \times 180^0$

$= 6 \times 180^0$

$= 1\,080^0$

Since the polygon is regular, all the sides and all the angles are equal.

Thus, one interior angle of a regular octagon $= \frac{1\,080^0}{8}$

$= 135^0$

Example 4

Find the number of sides of a regular polygon if one interior angle is 156^0.

Solution

One interior angle $= 156^0$
Sum of interior angles $= (n - 2) \times 180^0$

Since it is a regular polygon, all the sides and all the angles are equal.
So, one interior angle $= \dfrac{(n-2) \times 180^0}{n}$

$$\frac{(n-2) \times 180^0}{n} = 156^0$$

$$(n - 2) \times 180^0 = 156^0 \times n$$

$$180^0 n - 360^0 = 156^0 n$$

$$180^0 n - 156^0 n = 360^0$$

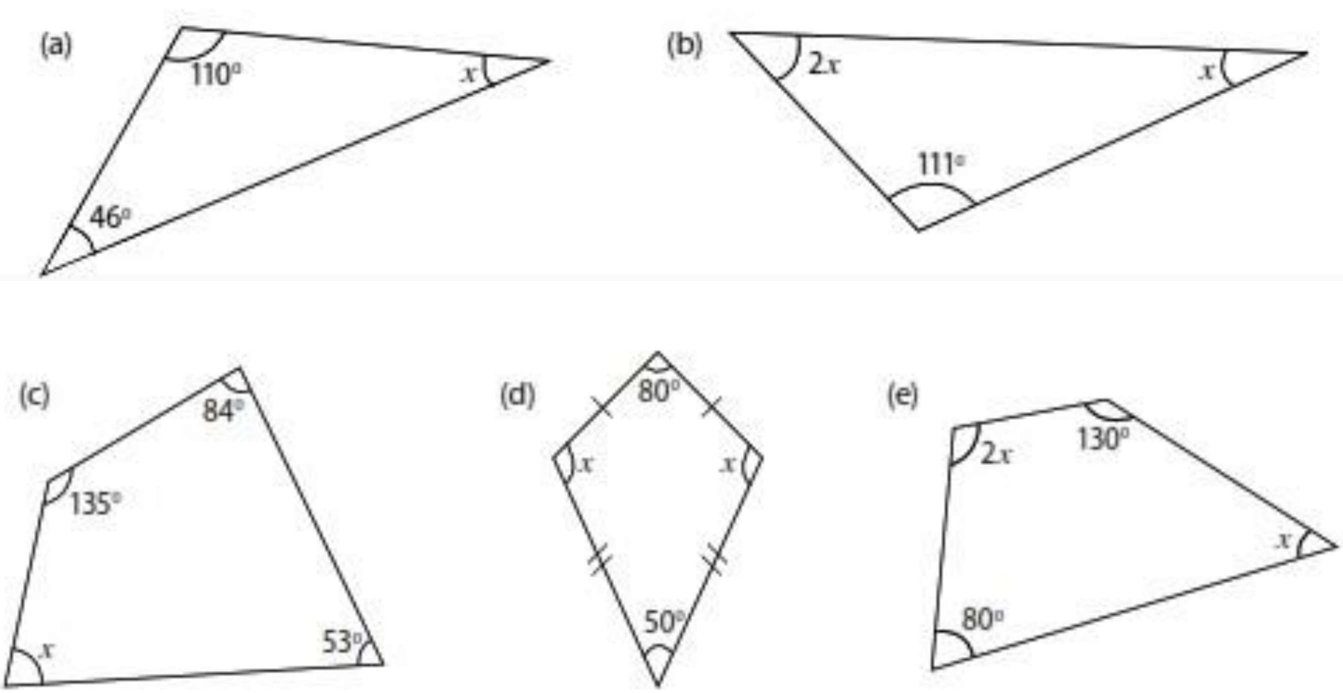
$$24^0 n = 360^0$$

$$n = \frac{360^0}{24^0} = 15$$

Hence, the number of sides of the regular polygon is 15.

Exercise: Workout all question.

2. In each of the following figures (not drawn to scale), find the value of x.



Sum of Exterior Angle

Example 1

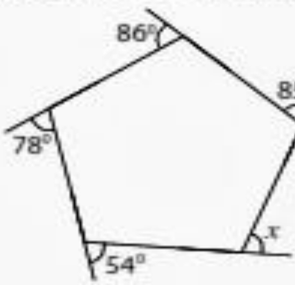
Find the sum of the exterior angles of a polygon with 25 sides.

Solution

Sum of exterior angles = 360^0

Example 2

Find the value of x in the diagram.



Solution

Sum of the exterior angles of a polygon = 360^0

$78^0 + 86^0 + 85^0 + 54^0 + x = 360^0$

$303^0 + x = 360^0$

$x = 360^0 - 303^0 = 57^0$

Example 3

Find the size of an exterior angle of a regular nonagon.

Solution

Sum of exterior angles of a nonagon = 360^0

Number of sides of a nonagon = 9

Size of an exterior angle = $\frac{360^0}{9} = 40^0$. *(Since it is a regular nonagon)*

Example 4

Two exterior angles of an hexagon are 40^0 and 36^0 while the remaining exterior angles are equal. Find one of the remaining exterior angles.

Solution

Number of sides of an hexagon = 6

Sum of exterior angles = 360^0

Let each of the remaining exterior angles be x. Number of remaining exterior angles = 4

$40^0 + 36^0 + 4x = 360^0$

$76^0 + 4x = 360^0$

$4x = 360^0 - 76^0$

$4x = 284^0$

$x = \frac{284^0}{4} = 71^0$

One of the remaining exterior angles is 71^0.

Example 5

Find the number of sides of a regular polygon having an interior angle of 144^0.

Solution

Method 1:

Each interior angle = 144^0

1 interior angle + 1 exterior angle = 180^0

Thus 1 exterior angle = $180^0 - 144^0 = 36^0$

Sum of exterior angles of any polygon = 360^0

Hence, number of sides = $\frac{360^0}{36^0} = 10$

Method 2:

One interior angle = 144^0

Sum of the interior angles = $(n-2) \times 180^0$

Since it is a regular polygon, all the sides and all the angles are equal.

One interior angle = $\frac{(n-2) \times 180^0}{n}$

$$\frac{(n-2) \times 180^0}{n} = 144^0$$

$$(n-2) \times 180^0 = 144^0 \times n$$

$$180^0 n - 360^0 = 144^0 n$$

$$180^0 n - 144^0 n = 360^0$$

$$36^0 n = 360^0$$

$$n = \frac{360^0}{36^0} = 10$$

Hence, the number of sides of the regular polygon is 10.

For a **regular** polygon with n sides, one interior angle = $\frac{(n-2) \times 180^0}{n}$

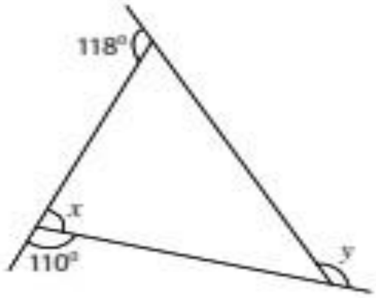
one exterior angle = $\frac{360^0}{n}$

Number of sides of a regular polygon given that the exterior angle is x^0 is $\frac{360^0}{x^0}$

Exercise: Workout all questions

2. Find the value of the unknown angles.

(a)



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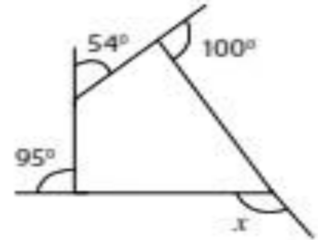

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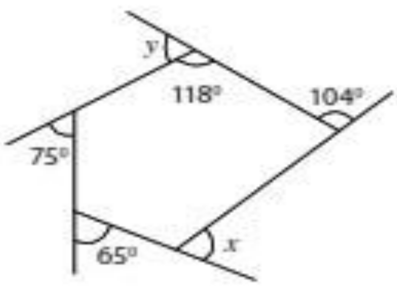

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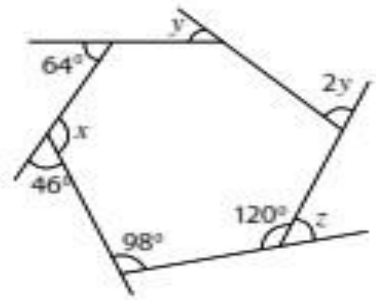

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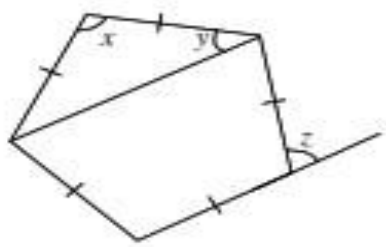

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